

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Numerical Value	Daily Tutorial Sheet - 12
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141.(7) Given equation is $4x^2 - 3x + \alpha = 0$

$\sin A$ and $\cos A$ are its roots, then $\sin A + \cos A = \frac{3}{4}$

$$\sin A \cos A = \frac{\alpha}{4}$$

Now, $\sin A + \cos A + \tan A + \cot A + \sec A + \operatorname{cosec} A = 7$

$$(\sin A + \cos A) + \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} + \frac{1}{\cos A} + \frac{1}{\sin A} = 7$$

$$(\sin A + \cos A) + \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} + \frac{\sin A + \cos A}{\sin A \cos A} = 7$$

$$\Rightarrow \frac{3}{4} + \frac{1 + (\sin A + \cos A)}{\sin A \cos A} = 7 \Rightarrow \frac{1 + \frac{3}{4}}{\frac{\alpha}{4}} = 7 - \frac{3}{4} = \frac{25}{4} \Rightarrow \frac{7}{\alpha} = \frac{25}{4} \therefore \frac{25\alpha}{4} = 7$$

142.(2) We have, $2k - 4 = 0 \therefore k = 2$

Now, $\sin x + \operatorname{cosec} x = 2$

$$\sin x + \frac{1}{\sin x} = 2$$

$$\sin^2 x + 1 = 2 \sin x$$

$$\sin^2 x - 2 \sin x + 1 = 0$$

$$(\sin x - 1)^2 = 0$$

$$\sin x - 1 = 0 \therefore \sin x = 1$$

Then, $\operatorname{cosec} x = 1 \left(\because 0 < x \leq \frac{\pi}{2} \right)$

So, $\sin^n x + \operatorname{cosec}^n x = (1)^n + (1)^n = 2 \quad (\because n \in \mathbb{N})$

143.(4) Given, $\tan(\pi \cos \theta) = \cot(\pi \sin \theta)$

$$\tan(\pi \cos \theta) = \tan \left[\pm \frac{\pi}{2} - \pi \sin \theta \right]$$

$$\pi \cos \theta = \pm \frac{\pi}{2} - \pi \sin \theta$$

$$\cos \theta = \pm \frac{1}{2} - \sin \theta$$

$$\sin \theta + \cos \theta = \pm \frac{1}{2}$$

$$\frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta = \pm \frac{1}{2\sqrt{2}}$$

$$\cos \theta \cos \frac{\pi}{4} + \sin \theta \sin \frac{\pi}{4} = \pm \frac{1}{2\sqrt{2}}$$

$$\cos\left(\theta - \frac{\pi}{4}\right) = \pm \frac{1}{2\sqrt{2}}$$

$$\pm \frac{1}{\lambda\sqrt{\mu}} = \pm \frac{1}{2\sqrt{2}} \quad (\text{Given})$$

$$\Rightarrow \lambda = 2, \mu = 2 \quad \therefore \lambda + \mu = 4$$

144.(1) Given, equation is $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos \beta - \sin \beta = 0$

Now, $S_1 = \tan \theta_1 + \tan \theta_2 + \tan \theta_3 + \tan \theta_4 = \sin 2\beta$

$$S_2 = \sum \tan \theta_1 \tan \theta_2 = \cos 2\beta$$

$$S_3 = \sum \tan \theta_1 \tan \theta_2 \tan \theta_3 = \cos \beta$$

$$S_4 = \tan \theta_1 \tan \theta_2 \tan \theta_3 \tan \theta_4 = -\sin \beta$$

We know that, $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4) = \frac{S_1 - S_3}{1 - S_2 + S_4}$

$$= \frac{\sin 2\beta - \cos \beta}{1 - \cos 2\beta - \sin \beta} = \frac{2 \sin \beta \cos \beta - \cos \beta}{2 \sin^2 \beta - \sin \beta} = \frac{\cos \beta [2 \sin \beta - 1]}{\sin \beta [2 \sin \beta - 1]} = \frac{\cos \beta}{\sin \beta} = \cot \beta$$

But $\tan(\theta_1 + \theta_2 + \theta_3 + \theta_4) = \cot(n\beta) \quad \therefore n = 1$

145.(0) $\therefore a \sin \theta = b \sin\left(\theta + \frac{2\pi}{3}\right) = c \sin\left(\theta + \frac{4\pi}{3}\right) = \frac{1}{k}$ (let)

Then, $\frac{1}{a} = k \sin \theta, \frac{1}{b} = k \sin\left(\theta + \frac{2\pi}{3}\right), \frac{1}{c} = k \sin\left(\theta + \frac{4\pi}{3}\right)$

$$\therefore \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = k \left[\sin \theta + \sin\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta + \frac{4\pi}{3}\right) \right]$$

$$\Rightarrow \frac{bc + ca + ab}{abc} = k \left[\sin \theta + 2 \sin\left(\theta + \pi\right) \cos \frac{\pi}{3} \right] \Rightarrow \frac{ab + bc + ca}{abc} = k \left[\sin \theta - 2 \sin \theta \cdot \frac{1}{2} \right]$$

$$\Rightarrow \frac{ab + bc + ca}{abc} = k(\sin \theta - \sin \theta) = 0$$

$$\therefore ab + bc + ca = 0$$

146.(2) LHS = $\frac{\cos 6x + 6 \cos 4x + 15 \cos 2x + 10}{\cos 5x + 5 \cos 3x + 10 \cos x} = \frac{(\cos 6x + \cos 4x) + 5 \cos 4x + 5 \cos 2x + 10 \cos 2x + 10}{\cos 5x + 5 \cos 3x + 10 \cos x}$

$$= \frac{2 \cos 5x \cdot \cos x + 5(\cos 4x + \cos 2x) + 10(1 + \cos 2x)}{\cos 5x + 5 \cos 3x + 10 \cos x} = \frac{2 \cos 5x \cdot \cos x + 10 \cos 3x \cdot \cos x + 20 \cos^2 x}{\cos 5x + 5 \cos 3x + 10 \cos x}$$

$$= \frac{2 \cos x [\cos 5x + 5 \cos 3x + 10 \cos x]}{(\cos 5x + 5 \cos 3x + 10 \cos x)} = 2 \cos x$$

147.(4) $\cos \alpha + \cos \beta + \cos \gamma + \cos(\alpha + \beta + \gamma) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right) + 2 \cos\left(\frac{\alpha + \beta + 2\gamma}{2}\right) \cos\left(\frac{\alpha + \beta}{2}\right)$

$$= 2 \cos\left(\frac{\alpha + \beta}{2}\right) \left[\cos\left(\frac{\alpha - \beta}{2}\right) + \cos\left(\frac{\alpha + \beta + 2\gamma}{2}\right) \right] = 4 \cos\left(\frac{\alpha + \beta}{2}\right) \left[2 \cos\left(\frac{2\alpha + 2\gamma}{4}\right) \cos\left(\frac{2\beta + 2\gamma}{4}\right) \right]$$

$$= 4 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha + \gamma}{2}\right) \cos\left(\frac{\beta + \gamma}{2}\right) \quad \therefore \quad \lambda = 4$$

$$148.(2) \quad F(\theta) = \frac{1}{\sin^2 \theta + 3 \sin \theta \cos \theta + 5 \cos^2 \theta} = \frac{1}{(\sin^2 \theta + \cos^2 \theta) + \frac{3}{2} \sin 2\theta + 4 \cos^2 \theta}$$

$$= \frac{1}{1 + \frac{3}{2} \sin 2\theta + 4 \cos^2 \theta} = \frac{2}{2 + 3 \sin 2\theta + 4 + 4 \cos 2\theta} = \frac{2}{3 \sin 2\theta + 4 \cos 2\theta + 6}$$

$$\therefore \quad F(\theta)_{\max} = \frac{2}{6-5} = \frac{2}{1} = 2 \quad (\because \text{minimum value of } 3 \sin 2\theta + 4 \cos 2\theta \text{ is } -5)$$

$$149.(1) \quad \therefore \quad \frac{\cos \theta_1}{\cos \theta_2} + \frac{\sin \theta_1}{\sin \theta_2} = 1 \quad \text{and} \quad \frac{\cos \theta_0}{\cos \theta_2} + \frac{\sin \theta_0}{\sin \theta_2} = 1$$

$\therefore \quad \theta_1$ and θ_0 be the roots of

$$\frac{\cos \theta}{\cos \theta_2} + \frac{\sin \theta}{\sin \theta_2} = 1 \quad \dots (i)$$

$$\Rightarrow \quad \frac{\cos \theta}{\cos \theta_2} = 1 - \frac{\sin \theta}{\sin \theta_2}$$

On squaring both sides, we get :

$$\Rightarrow \quad \frac{\cos^2 \theta}{\cos^2 \theta_2} = 1 + \frac{\sin^2 \theta}{\sin^2 \theta_2} - \frac{2 \sin \theta}{\sin \theta_2} \quad \Rightarrow \quad \frac{1}{\cos^2 \theta_2} - \frac{\sin^2 \theta}{\cos^2 \theta_2} = 1 + \frac{\sin^2 \theta}{\sin^2 \theta_2} - \frac{2 \sin \theta}{\sin \theta_2}$$

$$\Rightarrow \quad \left(\frac{1}{\cos^2 \theta_2} + \frac{1}{\sin^2 \theta_2} \right) \sin^2 \theta - \left(\frac{2}{\sin \theta_2} \right) \sin \theta + 1 - \frac{1}{\cos^2 \theta_2} = 0$$

$$\Rightarrow \quad \left(\frac{1}{\sin^2 \theta_2 \cos^2 \theta_2} \right) \sin^2 \theta - \left(\frac{2}{\sin \theta_2} \right) \sin \theta - \frac{\sin^2 \theta_2}{\cos^2 \theta_2} = 0$$

$\therefore \quad \theta_1$ and θ_0 be the roots of above equation, then product of roots

$$\sin \theta_1 \sin \theta_0 = - \frac{\frac{\sin^2 \theta_2}{\cos^2 \theta_2}}{\frac{1}{\sin^2 \theta_2 \cos^2 \theta_2}} = - \frac{\sin^2 \theta_2}{\cos^2 \theta_2} \times \frac{\sin^2 \theta_1 \cos^2 \theta_2}{1} = - \sin^2 \theta_2 \sin^2 \theta_1$$

$$\therefore \quad \frac{\sin \theta_1 \sin \theta_0}{\sin^2 \theta_2} = - \sin^2 \theta_2 \quad \dots (i)$$

Similarly, again taking equation (i)

$$\frac{\cos \theta}{\cos \theta_2} + \frac{\sin \theta}{\sin \theta_2} = 1 \quad \Rightarrow \quad \frac{\sin \theta}{\sin \theta_2} = 1 - \frac{\cos \theta}{\cos \theta_2}$$

$$\therefore \quad \frac{\cos \theta_1 \cos \theta_0}{\cos^2 \theta_2} = - \cos^2 \theta_2 \quad \dots (ii)$$

On adding equations (i) and (ii), we get :

$$\frac{\cos \theta_1 \cos \theta_0}{\cos^2 \theta_2} \frac{\sin \theta_1 \sin \theta_0}{\sin^2 \theta_2} = -(\sin^2 \theta_2 + \cos^2 \theta_2) = -1 \quad \therefore \left| \frac{\cos \theta_1 \cdot \cos \theta_0}{\cos^2 \theta_2} + \frac{\sin \theta_1 \cdot \sin \theta_0}{\sin^2 \theta_2} \right| = 1$$

$$\begin{aligned} 150.(6) \quad \therefore \quad \sin^3 x \sin 3x &= \frac{1}{4}(4 \sin^3 x) \sin 3x = \frac{1}{4}(3 \sin x - \sin 3x) \sin 3x \\ &= \frac{1}{4} \cdot \frac{1}{2}(2 \times 3 \sin 3x \cdot \sin x - 2 \sin^2 3x) = \frac{1}{8}[3(\cos 2x - \cos 4x) - (1 - \cos 6x)] \\ &= \frac{1}{8}[\cos 6x - 3 \cos 4x + 3 \cos 2x - 1] \text{ and } \sum_{m=0}^n C_m \cos nx = \sin^3 x \sin 3x \text{ is an identity in } x \\ &= \frac{1}{8}[\cos 6x - 3 \cos 4x + 3 \cos 2x - 1] \end{aligned}$$

On comparing both sides, we get : $n = 6$

$$\begin{aligned} 151.(0) \quad \therefore \quad \alpha \text{ is the common root of equation } x^2 - ax + 12 &= 0 \\ x^2 - bx + 15 &= 0 \text{ and } x^2 - (a+b)x + 36 = 0, \text{ then} \\ \alpha^2 - a\alpha + 12 &= 0 \quad \dots (i) \\ \alpha^2 - b\alpha + 15 &= 0 \quad \dots (ii) \\ \alpha^2 - (a+b)\alpha + 36 &= 0 \quad \dots (iii) \end{aligned}$$

On solving equations (i) and (ii), we get : $\alpha = \pm 3$

$$\alpha = 3 \quad (\because \alpha > 0)$$

Thus, $\cos x + \cos 2x + \cos 3x = \alpha = 3$ which is possible only when

$$\cos x = 1, \cos 2x = 1 \text{ and } \cos 3x = 1$$

Then, $\sin x = 0, \sin 2x = 0$ and $\sin 3x = 0$

$$\therefore \sin x + \sin 2x + \sin 3x = 0$$

$$152.(5) \text{ We have, } \frac{\lambda}{2} = 1 + 2(1 - \cos \theta) + 3(1 - \cos \theta)^2 + 4(1 - \cos \theta)^3 + \dots \infty \quad \dots (i)$$

We know that, $1 + 2x + 3x^2 + 4x^3 + \dots \infty = (1 - x)^{-2}$

Put $x = 1 - \cos \theta$, we get :

$$1 + 2(1 - \cos \theta) + 3(1 - \cos \theta)^2 + 4(1 - \cos \theta)^3 + \dots \infty = (1 - 1 + \cos \theta)^{-2} = \sec^2 \theta = 1 + \tan^2 \theta = 1 + \frac{3}{2} = \frac{5}{2}$$

$$\left(\because \tan \theta = \sqrt{\frac{3}{2}} \right)$$

$$\text{Now, } \frac{\lambda}{2} = \frac{5}{2} \quad [\text{From equation (i)}]$$

$$\therefore \lambda = 5$$

$$\begin{aligned} 153.(6) \quad \therefore \quad 3 \sin^2 x - 7 \sin x + 2 &= 0 \\ 3 \sin^2 x - 6 \sin x - \sin x + 2 &= 0 \end{aligned}$$

$$(\sin x - 2)(3 \sin x - 1) = 0$$

Either $\sin x - 2 = 0$

$$\Rightarrow \sin x = 2 \quad (\text{Not possible})$$

or $3 \sin x - 1 = 0$

$$\sin x = \frac{1}{3} = +ve$$

$\therefore \sin x$ is +ve in I and II quadrants.

Now, $[0, 2\pi] \rightarrow 2$ values in I and II quadrants

$(2\pi, 4\pi] \rightarrow 2$ values in I and II quadrants

$(4\pi, 5\pi] \rightarrow 2$ values in I and II quadrants

$\therefore$ Total number of values are 6.

154.(2) Given, $4 \operatorname{cosec}^2[\pi(p+t)] + p^2 - 4p = 0$

$$4 \operatorname{cosec}^2[\pi(p+t)] = 4p - p^2$$

$$\sin^2[\pi(p+t)] = \frac{4}{4p - p^2}$$

We know that, $0 \leq \sin^2[\pi(p+t)] \leq 1$

$$0 \leq \frac{4}{4p - p^2} \leq 1$$

If $\frac{4}{4p - p^2} \geq 0 \Rightarrow 4p - p^2 > 0 \Rightarrow p(-4 + p) < 0$

$$\therefore 0 < p < 4 \quad \dots \text{(i)}$$

If $\frac{4}{4p - p^2} \leq 1 \Rightarrow \frac{4 - 4p + p^2}{4p - p^2} \leq 0$

$$\Rightarrow \frac{(p-2)^2}{p(4-p)} \leq 0$$

$\frac{+}{-}$
 $\frac{+}{-}$

$\frac{+}{-}$
 $\frac{+}{-}$

$\frac{+}{-}$
 $\frac{+}{-}$

$$\therefore p \in (-\infty, 0) \cup \{2\} \cup (4, \infty) \quad \dots \text{(ii)}$$

Now, on taking common from equations (i) and (ii), we get :

$$p = 2$$

155.(2) We have, $\sum_{r=1}^5 \cos rx = 5$ in $[0, 2\pi]$

$$\cos x + \cos 2x + \cos 3x + \cos 4x + \cos 5x = 5$$

It is possible only when $\cos x = \cos 2x = \cos 3x = \cos 4x = \cos 5x = 1$ ($\because -1 \leq \cos \theta \leq 1$)

and it is satisfied by $x = 0, 2\pi$

$\therefore$ Number of solutions is 2.